WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

MTMADSE01T-MATHEMATICS (DSE1/2)

LINEAR PROGRAMMING

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any *five* questions from the following: $2\times 5 = 10$

(a) Prove that the vectors $(1,1,0)$, $(0,1,1)$ and $(1,2,1)$ form a basis in E^3.

(b) Check whether $x=5$, $y=0$, $z=-1$ is a basic solution of the system of equations:

$$x+2y+z=4,$$
$$2x+y+5z=5$$

(c) If $C(X)=\{(x,y):|x|\le 2, |y|\le 1\}$ be a convex hull then find set X.

(d) Find graphically the feasible space, if any, of the following:

$$x_1+2x_2\ge 2$$
$$5x_1+3x_2\le 15,\ x_1, x_2\ge 0$$

(e) Define fair game and strictly determinate game.

(f) Find the maximum number of possible way of assignment of a 5×5 assignment problem.

(g) What is the criterion for no feasible solution in two-phase method?

(h) Define saddle point. Find the value of the game of the pay-off matrix

Player Q

Player P		B_1	B_2
	A_1	1	–1
	A_2	–1	1

2. A business manager has the option of investing money in two plans. Plan A guarantees that each rupee invested will earn 70 paise a year and plan B guarantees that each rupee invested will earn Rs. 2.00 every two years. In plan B, only investments for periods that are multiples of 2 years are allowed. How should the manager invest Rs. 50,000/- to maximize the earnings at the end of 3 years? Formulate the problem as a Linear Programming Problem with two legitimate variable. Find the optimum solution using graphical method. 4+4

3. State and prove fundamental theorem of LPP. 8

4. Use Two Phase method to solve the following linear programming problem: 8

Maximize $z = 2x_1 + x_2 + x_3$

Subject to $4x_1 + 6x_2 + 3x_3 \le 8$

$3x_1 - 6x_2 - 4x_3 \le 1$

$2x_1 + 3x_2 - 5x_3 \ge 4$

$x_1, x_2, x_3 \ge 0$

5. (a) Prove that the set of all convex combination of a finite number of points is a convex. 4

(b) Reduce the feasible solution $(1, 2, 1)$ of the following system of equation to a basic feasible solution. 4

$x_1 - x_2 + 2x_3 = 1$

$x_1 + 2x_2 - x_3 = 4$

6. State and prove fundamental theorem of duality. 8

7. Solve the following LPP using duality theory: 8

Minimize $z = x_1 + x_2 + x_3$

Subject to $x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_2 \le 3$

$2x_2 - x_3 \ge 4$

$x_1, x_2 \ge 0$ and x_3 is unrestricted in sign.

8. (a) Find the optimal assignment and the corresponding assignment cost for the assignment problem with the following cost matrix: 4

	D_1	D_2	D_3	D_4	D_5
O_1	2	4	3	5	4
O_2	7	4	6	8	4
O_3	2	9	8	10	4
O_4	8	6	12	7	4
O_5	2	8	5	8	8

(b) Find the initial B.F.S. of the following transportation problem by VAM method hence find the optimal solution: 4

	D_1	D_2	D_3	D_4	a_j
O_1	19	14	23	11	11
O_2	15	16	12	21	13
O_3	30	25	16	39	18
b_j	6	10	11	15	

9. Prove that the mixed strategies p^*, q^* will be optimal strategy of the game if and only if $E(p, q^*) \le E(p^*, q^*) \le E(p^*, q)$ 8

10.(a) Solve graphically the following game problem: 5

		B_1	B_2
A	A_1	2	7
	A_2	3	5
	A_3	11	2

(b) Use dominance method to reduce the payoff matrix in a 2×2 game. Hence solve it. 3

	B_1	B_2	B_3
A_1	8	5	8
A_2	8	6	5
A_3	7	4	5
A_4	6	5	6

11. In a rectangular game, the pay-off matrix is given by 8

		Player Q		
		Q_1	Q_2	Q_3
Player P	P_1	3	2	–1
	P_2	4	0	5
	P_3	–1	3	–2

State with justification, whether the players will choose pure or mixed strategies. Solve the game problem by converting it into a L.P.P.

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